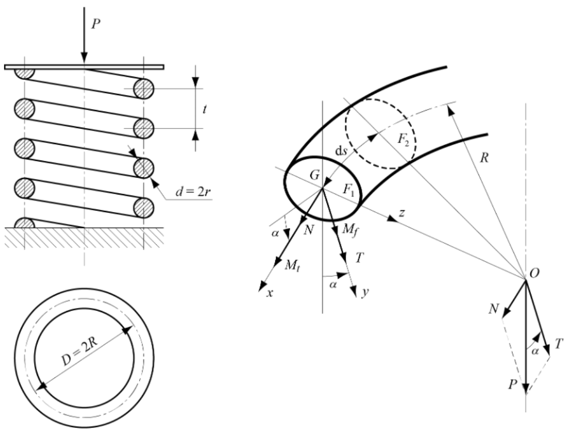
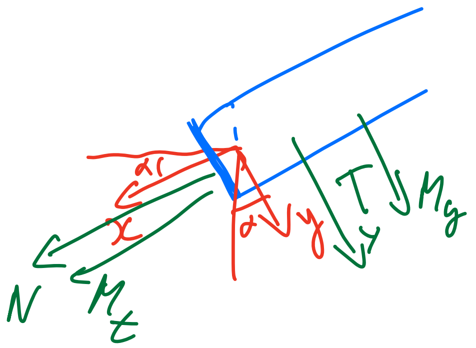
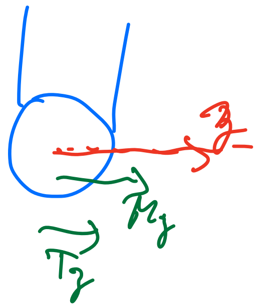
5.2





$$\vec{P} = P\sin\alpha\,\vec{e_x} + P\cos\alpha\,\vec{e_y}$$

$$\sum F_x = 0 \rightarrow \quad N - P\sin\alpha = 0 \qquad N = -P\sin\alpha$$

$$\sum F_y = 0 \qquad T + P\cos\alpha = 0 \qquad T = -P\cos\alpha$$

$$\sum F_z = 0 \qquad T_z = 0$$

$$\sum \vec{M_G} = 0 = M_y\,\vec{e_y} + M_z\,\vec{e_z} + \vec{GO} \times \vec{P}$$

$\hookrightarrow -R\,\vec{e_z}$

$M_y = PR \sin\alpha \qquad M_z = 0 \qquad x\,;\ M_t = PR\cos\alpha$

$$\frac{\partial M_f}{\partial P} = R\sin\alpha \qquad \frac{\partial M_t}{\partial P} = R\cos\alpha$$

## 2. EFFET DE TORSION SUR DEFORMÉE VERTICALE

$$\delta_T = \frac{\partial U_T}{\partial P} = \int_{x=0}^{x=L=nD\pi} \frac{M_t}{G I_p}\frac{\partial M_t}{\partial P}\,dx$$

$$= \int_{x=0}^{nD\pi} \frac{PR^2}{G I_p} \cos^2\alpha \, dx \qquad \simeq 1$$

$I_p \rightarrow \pi d^4/32$

$$\delta_T = \frac{8nPD^3}{G d^4} \qquad \text{unités } \frac{N.\,m^3\,m^2}{N \quad m^4} = [m] \checkmark$$

$> 0$

## 3 EFFET de FLEXION

$$\delta_F = \frac{\partial U_F}{\partial P} = \int_{x=0}^{n\pi D} \frac{M_f}{EI}\frac{\partial M_f}{\partial P}\,dx$$

$$= \int_{x=0}^{n\pi D} \frac{PR^2}{EI} \sin^2\alpha \, dx \quad = \alpha^2$$

$$I = \frac{\pi d^4}{64}$$

$$\delta_F = \frac{PD^2}{E\pi d^4} \frac{64}{4} (n\pi D) \left(\frac{t}{2\pi R}\right)^2$$

$$= \frac{16n \, Pt^2 D}{E d^4 \pi^2}$$

unités $\frac{N \, m^3 m^2}{N \, m^4} = [m]$

TD

$$\boxed{\delta = \delta_F + \delta_T = \frac{8nP}{d^4}\left(\frac{2t^2D}{E\pi^2} + \frac{D^3}{G}\right)}$$

flex torsion

$\frac{t^2 D}{E} \ll \frac{D^3}{G}$

$t \sim 1mm$ $\rightarrow 100x$

$D \sim 1cm$

$$k = \frac{P}{\delta} = \frac{d^4}{8n} \frac{1}{\left(\frac{2t^2D}{E\pi^2} + \frac{D^3}{G}\right)} \simeq \frac{d^4}{8n} \frac{G}{D^3}$$

$$U = P\delta \frac{1}{2} = \frac{1}{2} \frac{P^2}{k} = \frac{1}{2} k \delta^2$$

$$U = \frac{d^4}{8n}\,\frac{G}{D^3}\left(\frac{8n\,P D^3}{G d^4}\right)^2 = \frac{4n\,P^2 D^3}{G d^4}$$